

# Anomalous Josephson effect in $p$ -wave dirty junctions

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The Josephson effect in  $p$ -wave superconductor / diffusive normal metal /  $p$ -wave superconductor junctions is studied theoretically. Amplitudes of Josephson currents are several orders of magnitude larger than those in  $s$ -wave junctions. Current-phase ( $J$ - $\varphi$ ) relations in low temperatures are close to those in ballistic junctions such as  $J \propto \sin(\varphi/2)$  and  $J \propto \varphi$  even in the presence of random impurity potentials. A cooperative effect between the midgap Andreev resonant states and the proximity effect causes such anomalous properties and is a character of the spin-triplet superconductor junctions.

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The *internal*  $\pi$ -phase shift (sign change) of pair potentials is essential for unconventional superconductivity and is the source of the midgap Andreev resonant state (MARS)<sup>1,2,3</sup>. It is now known that the MARS is responsible for anomalous transport properties in superconducting junctions<sup>4</sup>. In normal metal/superconductor junctions, transport properties are affected also by the proximity effect which is interpreted in terms of diffusion of Cooper pairs into normal metals. In what follows, we assume that normal metals are in the diffusive transport regime due to impurity scatterings. Recent theoretical studies have revealed sensitivity of the proximity effect to the *internal* phase of pair potentials<sup>5,6</sup>. In normal metals attached to unconventional superconductors, Cooper pairs have a sign degree of freedom reflecting the  $\pi$ -phase shift of pair potentials. Suppression of the proximity effect is usually expected because wave function of a Cooper pair originated from the positive part of pair potentials cancel that originated from the negative part<sup>5,6</sup>. Two of us, however, discussed anomalous enhancement of the zero-bias tunneling conductance due to the proximity effect in a presence of the MARS<sup>7,8</sup>.

In superconductor / normal metal / superconductor junctions, another phase degree of freedom affects quantum transport. Namely, the *external* phase difference across the junctions  $\varphi$  drives Josephson currents. An importance of studying the Josephson effect is growing these days because quantum interference devices consisting of Josephson junctions can be basis of future technologies. In fact, a recent experiment has tried to apply high- $T_c$  superconductors to coherent devices<sup>9</sup>. In unconventional junctions, the MARS is considered to have the phase degree of freedom. When MARS's are formed at the two junction interfaces, the external phase may modify interference effects between the two MARS's and Josephson currents. The research in this direction can shed new light on quantum transport in unconventional superconductors.

In this paper, we theoretically study Josephson currents between two  $p$ -wave superconductors through nor-

mal metals by solving the Bogoliubov-de Gennes equation using the recursive Green function method<sup>10,11</sup>. We show that amplitudes of Josephson currents in the  $p$ -wave junctions are much larger than those in the  $s$ -wave junctions when transmission probabilities of junction interfaces are small. The local density of states in normal metals has a zero-energy peak reflecting anomalous diffusion of the MARS's into a normal metal and that spatial profiles of the zero-energy peak depend strongly on  $\varphi$ . As a consequence, current-phase ( $J$ - $\varphi$ ) relations remarkably deviate from the sinusoidal function in low temperatures and are close to those in ballistic junctions such as  $J \propto \sin(\varphi/2)$  and  $J \propto \varphi$ <sup>12,13,14,15</sup>. The resonant tunneling through the MARS in normal metals is responsible for such unusual Josephson effect. The obtained results imply high potentials of spin-triplet superconducting junctions as coherent devices.

We consider three pairing symmetries on two dimensional superconductors: (i)  $\Delta_{\mathbf{k}} = \Delta_0$  for  $s$ -wave, (ii)  $\Delta_0 2\bar{k}_x \bar{k}_y$  for  $d_{xy}$ -wave, and (iii)  $\Delta_0 \bar{k}_x$  for  $p_x$ -wave symmetries. Here  $\Delta_0$  is the maximum amplitude of pair potentials at the zero temperature,  $\bar{k}_x = k_x/k_F$  and  $\bar{k}_y = k_y/k_F$  are normalized wave numbers on the Fermi surface in the  $x$  and  $y$  directions, respectively. Josephson currents are parallel to the  $x$  direction and junction interfaces are parallel to the  $y$  direction as shown in Fig. 1(a). The pair potentials in momentum space are illustrated in Fig. 1(b). An interference of a quasiparticle enables formation of the MARS at a junction interface when  $\Delta_{\bar{k}_x, \bar{k}_y} \Delta_{-\bar{k}_x, \bar{k}_y} < 0$ <sup>4,16</sup>. The pair potentials in the  $d_{xy}$ - and  $p_x$ -symmetries satisfy the relation for all wave numbers. The absence of the proximity effect in normal metals is described by a relation<sup>5,6</sup>  $\Delta_{\bar{k}_x, \bar{k}_y} = -\Delta_{\bar{k}_x, -\bar{k}_y}$ . The pair potential in the  $d_{xy}$ -symmetry satisfies the relation. Thus the proximity effect is expected in both the  $s$ - and  $p_x$ -wave symmetries. In Fig. 1(b), we classify the pairing symmetries into three groups by the presence (○) or absence (×) of the two interference effects<sup>7,8</sup>. Within  $p$ -wave symmetries, we pay special attention to the  $p_x$ -wave symmetry because the proximity effect and MARS

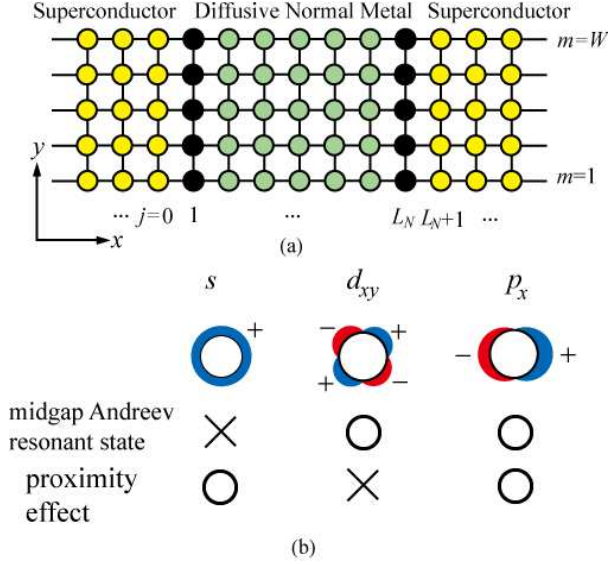


FIG. 1: A schematic figure of a Josephson junction on the tight-binding lattice is shown in (a). In (b), we illustrate the pair potentials in momentum space, where open circles represent the Fermi surface. The pair potentials are classified into three groups by the presence or absence of the two interference effects.

are present at the same time. On the other hand in the  $p_y$ -wave symmetry, neither is present<sup>7</sup>.

Let us consider Josephson junctions on the two-dimensional tight-binding model as shown in Fig. 1(a). A vector  $\mathbf{r} = j\mathbf{x} + m\mathbf{y}$  points a lattice site, where  $\mathbf{x}$  and  $\mathbf{y}$  are the unit vectors in the  $x$  and  $y$  directions, respectively. The junction consists of three regions: a normal metal (i.e.,  $1 \leq j \leq L_N$ ) and two superconductors (i.e.,  $-\infty \leq j \leq 0$  and  $L_N + 1 \leq j \leq \infty$ ). In the  $y$  direction, the number of lattice sites is  $W$  and we assume the periodic boundary condition. Electronic states in superconducting junctions are described by the mean-field Hamiltonian

$$H_{\text{BCS}} = \frac{1}{2} \sum_{\mathbf{r}, \mathbf{r}'} \left[ \hat{c}_{\mathbf{r}}^{\dagger} \hat{h}_{\mathbf{r}, \mathbf{r}'} \tilde{c}_{\mathbf{r}'} - \overline{\tilde{c}_{\mathbf{r}}} \hat{h}_{\mathbf{r}, \mathbf{r}'}^* \overline{\hat{c}_{\mathbf{r}'}^{\dagger}} \right] + \frac{1}{2} \sum_{\mathbf{r}, \mathbf{r}' \in \text{S}} \left[ \hat{c}_{\mathbf{r}}^{\dagger} \hat{\Delta}_{\mathbf{r}, \mathbf{r}'} \overline{\tilde{c}_{\mathbf{r}'}} - \overline{\tilde{c}_{\mathbf{r}}} \hat{\Delta}_{\mathbf{r}, \mathbf{r}'}^* \tilde{c}_{\mathbf{r}'} \right], \quad (1)$$

$$\hat{h}_{\mathbf{r}, \mathbf{r}'} = [-t\delta_{|\mathbf{r}-\mathbf{r}'|,1} + (\epsilon_{\mathbf{r}} - \mu + 4t)\delta_{\mathbf{r}, \mathbf{r}'}] \hat{\sigma}_0 + \mathbf{V}(\mathbf{r} \cdot \hat{\boldsymbol{\sigma}}) \quad (2)$$

$$\hat{\Delta}_{\mathbf{r}, \mathbf{r}'} = i\Delta\hat{\sigma}_2, \quad (3)$$

$$\tilde{c}_{\mathbf{r}} = \begin{pmatrix} c_{\mathbf{r}, \uparrow} \\ c_{\mathbf{r}, \downarrow} \end{pmatrix}, \quad (4)$$

where  $\hat{c}_{\mathbf{r}, \sigma}^{\dagger}$  ( $c_{\mathbf{r}, \sigma}$ ) is the creation (annihilation) operator of an electron at  $\mathbf{r}$  with spin  $\sigma = (\uparrow \text{ or } \downarrow)$  and  $\overline{\tilde{c}}$  means the transpose of  $\tilde{c}$ . The hopping integral  $t$  is considered

among nearest neighbor sites. We assume that  $t$  and the Fermi energy  $\mu$  are common in superconductors and a normal metal. In a normal metal, on-site potentials are given randomly in the range of  $-V_I/2 \leq \epsilon_{\mathbf{r}} \leq V_I/2$ . We introduce insulating barriers at  $j = 1$  and  $L_N$ , where  $\epsilon_{\mathbf{r}}$  is given by  $V_B$ . Two superconductors in which  $\epsilon_{\mathbf{r}}$  are taken to be zero are identical to each other. In the  $p_x$ -wave symmetry, a spin vector of Cooper pairs  $\mathbf{d}$  points the  $z$  direction. The arguments below do not depend on directions of  $\mathbf{d}$ . The Hamiltonian is diagonalized by the Bogoliubov transformation and the Bogoliubov-de Gennes equation is numerically solved by the recursive Green function method<sup>10,11</sup>. Josephson currents are given by

$$J = -ietT \sum_{\omega_n} \text{Tr} \left[ \hat{G}_{\omega_n}(\mathbf{r}', \mathbf{r}) - \hat{G}_{\omega_n}(\mathbf{r}, \mathbf{r}') \right] \quad (5)$$

with  $\mathbf{r}' = \mathbf{r} + \mathbf{x}$ , where  $\hat{G}_{\omega_n}$  is the Green function and  $\omega_n = (2n + 1)\pi T$  is the Matsubara frequency with  $n$  and  $T$  being an integer and a temperature, respectively. In Eq. (5), Tr means the trace in the Nambu space and the summation over  $m$ . In this paper, the unit of  $\hbar = k_B = 1$  is used with  $k_B$  being the Boltzmann constant. Local density of states is also calculated from  $N(E, j) = -\text{Im} \text{Tr} \hat{G}_{E+i\gamma}(\mathbf{r}, \mathbf{r})/\pi$ , where  $E$  is measured from the Fermi energy and  $\gamma$  is a small imaginary part. Throughout this paper, we fix parameters as  $L_N = 70$ ,  $W = 25$ ,  $\mu = 2t$ , and  $V_I = 2t$ . Under these parameters, normal metals are in the diffusive transport regime, where the mean free path in normal metals is estimated about  $\ell \sim 6$  lattice constants and the Thouless energy  $E_{\text{th}}$  is calculated to be  $1.6 \times 10^{-3}t$ . Results discussed below are qualitatively insensitive to these parameters.

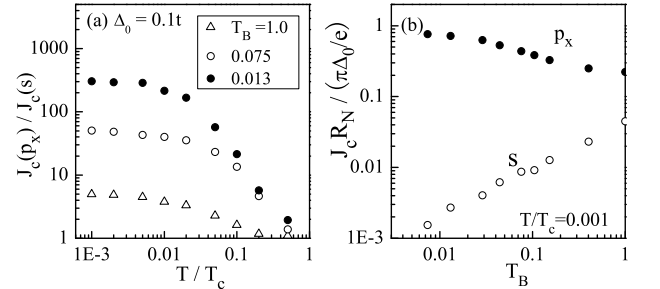


FIG. 2: The maximum amplitudes of Josephson currents in the  $p_x$ -wave symmetry  $J_c(p_x)$  are compared with those in the  $s$ -wave symmetry  $J_c(s)$  in (a), where  $T_B$  is the transmission probability of potential barriers in the normal states. In (b),  $J_c(p_x)$  and  $J_c(s)$  are plotted as a function of  $T_B$  at  $T = 0.001T_c$ .

At first we show that the maximum amplitudes of Josephson currents in the  $p_x$ -wave symmetry  $J_c(p_x)$  become much larger than those in the  $s$ -wave  $J_c(s)$ . In

Fig. 2(a), ratios  $J_c(p_x)/J_c(s)$  are plotted as a function of temperatures for  $\Delta_0 = 0.1t$ . Here we choose several values of the barrier potentials  $V_B$  at  $j = 1$  and  $L_N$ . The resulting normal transmission probabilities of the barrier  $T_B$  are 1.0, 0.075 and 0.013 for  $V_B/t = 0, 6$  and 15, respectively. The ratios  $J_c(p_x)/J_c(s)$  increase with decreasing  $T$  and amazingly become more than 100 in low temperatures for small  $T_B$ . The amplitudes of Josephson currents in the  $p_x$ -wave junctions are much larger than those in the  $s$ -wave junctions. In Fig. 2(b),  $J_c R_N$  normalized by  $\pi\Delta_0/e$  is plotted as a function of  $T_B$  at  $T = 0.001T_c$ , where  $R_N$  is the normal resistance of junctions. The results show that  $J_c R_N$  in the  $s$ -wave decreases with decreasing  $T_B$ , whereas that in the  $p_x$ -wave increases.

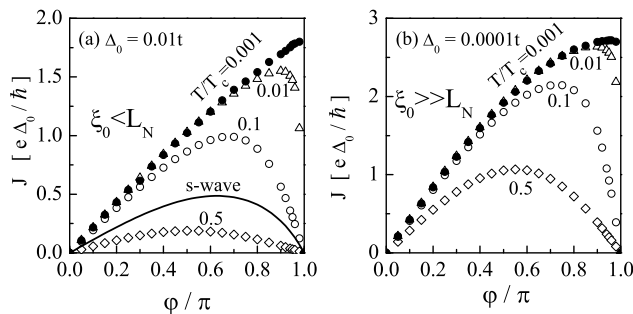


FIG. 3: Current-phase relations for the  $p_x$ -wave symmetries are shown for several temperatures at  $V_B = 0$ , where  $\Delta_0 = 0.01t$  in (a) and  $\Delta_0 = 0.0001t$  in (b). For comparison, results in the  $s$ -wave junctions at  $T = 0.001T_c$  is shown with a solid line in (a), where the amplitude of Josephson current is multiplied by 5.

We next focus on current-phase relations of the Josephson effect. In Fig. 3, Josephson currents are plotted as a function of  $\varphi$  for the  $p_x$ -wave symmetries at  $V_B = 0$ . Parameters are chosen as  $\Delta_0 = 0.01t$  and  $0.0001t$  in (a) and (b), respectively. The current-phase relations are almost sinusoidal function in a high temperature at  $T = 0.5T_c$ . At  $T = 0.001T_c$ , however, the current-phase relations are close to  $J \propto \varphi$  and  $J \propto \sin(\varphi/2)$  in (a) and (b), respectively. These are characteristic current-phase relations in ballistic Josephson junctions in the  $s$ -wave symmetry<sup>12,13,14</sup>. We have confirmed that these current-phase relations remain even in the presence of potential barriers (i.e.,  $V_B \neq 0$ ).

The results imply large contributions of the multiple Andreev reflection in low temperatures. In general, Josephson currents can be decomposed into a series of  $J = \sum_{n=1}^{\infty} J_n \sin(n\varphi)$ , where  $J_n$  for  $n \geq 2$  represent contributions of the multiple Andreev reflection. Roughly speaking,  $J_n$  is proportional to  $\{T_N\}^n$  with  $T_N$  being the transmission probability of a quasiparticle from the left superconductor to the right superconductor through

the normal segment (including two barriers and a normal metal). Thus the multiple Andreev reflection is negligible (i.e.,  $J_1 \gg J_2 \gg J_3 \dots$ ) for  $T_N \ll 1$ . On the other hand in the case of  $T_N = 1$ , the multiple Andreev reflection leads to the deviation of current-phase relations from the sinusoidal function. It is noted at  $T_N = 1$  that we obtain  $J \propto \varphi$  and  $J \propto \sin(\varphi/2)$  at the zero temperature for  $L_N \gg \xi_0$  and  $L_N \ll \xi_0$ , respectively<sup>12,13,14</sup>.

In Fig. 3(a), we also show the current-phase relations in the  $s$ -wave symmetry at  $T = 0.001T_c$  with a solid line. The current-phase relation in the  $s$ -wave is described almost by the sinusoidal function<sup>17</sup> because impurity potentials in normal metals suppress  $T_N$  and therefore the multiple Andreev reflection. In the  $p_x$ -wave junctions, the coherence length  $\xi_0$  are estimated about 50 lattice constants in (a) and 5000 in (b). Thus  $L_N > \xi_0$  and  $L_N \ll \xi_0$  are satisfied in (a) and (b), respectively. The current-phase relations such as  $J \propto \sin(\varphi/2)$  in (b) and  $J \propto \varphi$  in (a) are universal properties of the  $p_x$ -wave junctions in low temperatures because they are independent of the strength of barrier potentials and the degree of disorder in normal metals. The calculated results in Fig. 3 indicate  $T_N = 1$  even in the presence of impurity potentials. The large amplitudes of the Josephson current in Fig. 2 are also explained by  $T_N = 1$ .

The calculated results in Figs. 2 and 3 show the specific properties of Josephson currents in the  $p_x$ -wave junctions. In what follows, we analyze quasiparticle states in normal metals to understand the origin of the anomalous Josephson effect. In Fig. 4, we show the local density of states in normal metals for the  $s$ - and  $p_x$ -wave symmetries, where  $\Delta_0 = 0.005t$ ,  $\gamma = 0.05\Delta_0$ , and  $N_0$  denotes the normal density of states. At  $\varphi = 0$  in the  $s$ -wave junctions in (a), the local density of states for  $E < E_{th} \sim 0.3\Delta_0$  is suppressed because of the proximity effect. The suppression of the local density of states indicates the conversion of quasiparticles to Cooper pairs in normal metals. At  $\varphi = \pi$  in (b), the local density of states recovers its amplitude for  $E < E_{th}$ . The wave function of Cooper pairs from the left superconductor and that from the right one cancel each other around  $\varphi \sim \pi$  as schematically illustrated in a picture below the calculated results.

The local density of states is drastically changed in the  $p_x$ -wave symmetry as shown in (c) and (d). Zero-energy peaks whose width is determined by  $\gamma$  can be seen, which means formation of the midgap Andreev resonant state (MARS) in normal metals. Although the MARS originally localizes at junction interfaces<sup>4</sup>, the MARS penetrates into normal metals in the presence of the proximity effect. Spatial profiles of the local density of states depend remarkably on the external phase difference as shown in (c) and (d). At  $\varphi = 0$ , the zero-energy peak disappears at the center of normal metals ( $j \sim 35$ ) because wave function of the MARS from the left superconductors cancel out that from the right one as shown schematically in a lower panel in (c). On the other hand in (d), wave functions of the MARS in the two supercon-

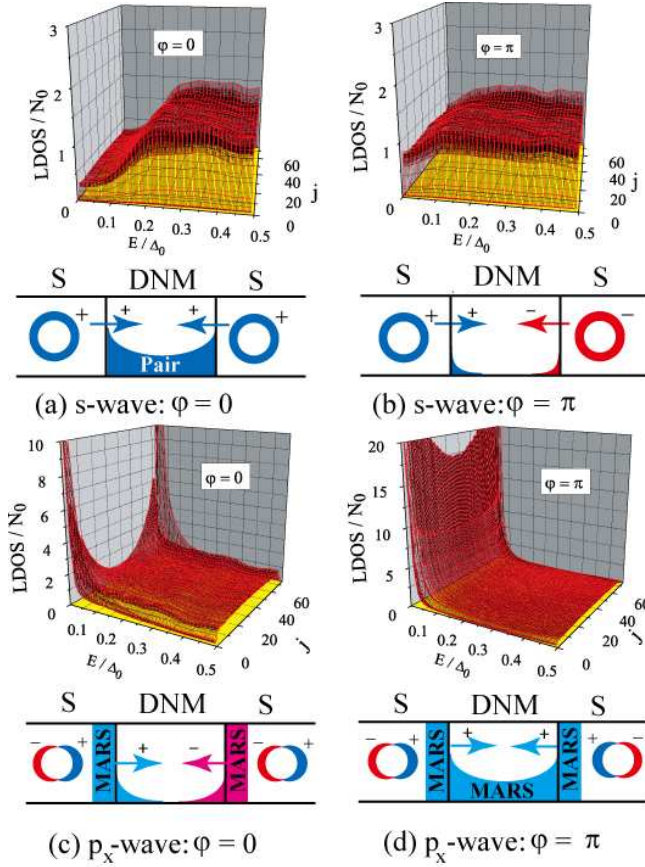


FIG. 4: Local density of states (LDOS) in normal metals ( $1 \leq j \leq L_N = 70$ ) are shown for the  $s$ -wave and  $p_x$ -wave symmetries. The left and right superconductors are attached at  $j = 0$  and  $j = 71$ , respectively. Noting that  $E_{th}$  is about  $0.3\Delta_0$ . In schematic pictures, DNM and S denote a diffusive normal metal and a superconductor, respectively. The local density of states shown here are calculated in the absence of Josephson currents. We have confirmed that the results at  $\varphi = 0.99\pi$  qualitatively shows the same behavior as those at  $\varphi = \pi$ .

ductors have the same sign with each other. Thus the two MARS's can penetrate deeply into normal metals and the zero-energy peak can be seen everywhere. We note that the penetration of the MARS is possible only when the proximity effect is present in normal metals. In fact, we have confirmed that no zero-energy peak is found in normal metals in the  $d_{xy}$ -wave symmetry (results are not shown) and that the ensemble average of Josephson currents vanishes because the proximity effect is absent in normal metals<sup>5</sup>. Fig. 4 indicates that the proximity effect bridges the two MARS's in the two superconductors. Thus  $T_N = 1$  holds because of the resonant transmission through the MARS in normal metals. The Josephson effect specific to the  $p_x$ -wave symmetry discussed in Figs. 2 and 3 are a consequence of the diffusion of the MARS into normal metals.

In summary, we found anomalous behaviors of Josephson currents in superconductor / normal metals / superconductor junctions in the  $p_x$ -wave symmetry. The maximum amplitudes of Josephson currents  $J_c$  in the  $p_x$ -wave junctions become much larger than those in the  $s$ -wave junctions. It is known that large values of  $J_c$  are desired in device applications because  $J_c R_N$  limits operation speeds of Josephson devices. Current-phase relations in low temperatures are close to those in ballistic junctions such as  $J \propto \sin(\varphi/2)$  and  $J \propto \varphi$  independent of the strength of potential barriers at interfaces and the degree of disorder in normal metals. The two the midgap Andreev resonant states penetrate deeply into normal metals, which causes the unusual Josephson effect in  $p_x$ -wave superconducting junctions. The anomalous Josephson effect is a novel feature of phase-sensitive transport in spin-triplet superconducting junctions.

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